Strongly Correlated Systems

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Most of the theeoretical methods and results found in physics textbooks deal with a scientific version of the "divide and conquer" paradigm: an apparently complicated system is approximated by simple building blocks, whose behavior can be found using fairly straightforward mathematical or numerical methods. Our overall picture of nature is deeply rooted in this way of solving problems.

However, many phenomena, which are both very interesting and potentially useful, cannot be understood this way. Loosely speaking, such phenomena are specific to strongly interacting, correlated systems, and the interplay between disorder, symmetry, and dimensional constraints. Whenever progress was made in the study of such problems (examples are BCS superconductivity, the general theory of phase transitions, the Quantum Hall effect), it had a tremendous scientific impact and required the development of significantly new analytical methods.

Along with great discoveries, the legacy of the last century includes open questions of this nature, whose number promises only to be increased by the new directions in physics research, like complex materials, communications, and biological systems. Developing new tools to tackle these challenges is likely to be a main theme for modern theoretical physics.

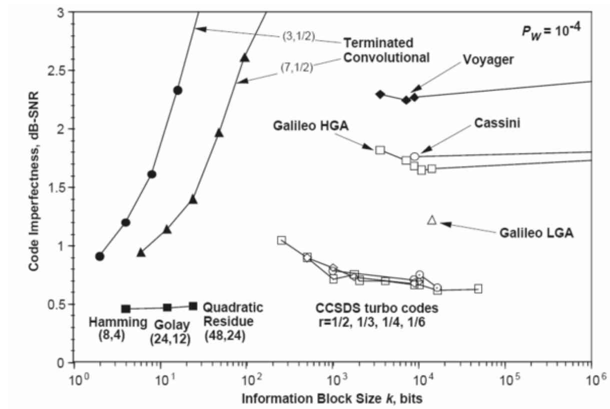

FIG. 1: NASA's Deep Space Missions ECC Codes

A. Complex systems and information theory

What do mobile phone networks, sub-micron semiconductor devices, and number theory have in common? The answer is a theoretical model based on graphs, which has the potential to explain elusive collective behavior responsible for "bad" configurations in coding algorithms used in networking, as well as for the transition of a 2D gas of electrons from a conductor to an insulator. In more abstract terms, it may also be related to the peculiar property of the Riemann zeta-function known as the *Riemann hypothesis* (one of the Millenium problems posed by the Clay Institute of Mathematics).

With Misha Chertkov (T-13) and Nandu Santhi (CNLS), we are investigating the application of concepts and methods from the physics of disorded media (supersymmetry, transfer matrix, Bethe anszatz), statistical inference (Bayesian networks, belief propagation algorithms), and algebraic structures (Reed-Solomon codes) to this class of problems [1] (see FIG. 1).

This set of problems has a considerable practical interest, because of the direct applications to secured communications and network detection, but also to open questions in the physics of disordered media.

B. Nonlinear dynamics of correlated systems

When two fluids of very different viscosities (like air and oil) are confined to a two dimensional geometry, with one being pumped in and the other out, all the dynamics of the system is encoded in the boundary between the two. For almost all initial conditions, this boundary will evolve into a singular limit, where sharp regions become even sharper, and eventually split into secondary fingers, leading to a self-similar structure of high complexity (FIG. 2). The mathematical model which describes this problem is termed Laplacian Growth. An identical complex structure is obtained as a result of purely *stochastic* growth model, in which random walkers released from a far boundary diffuse towards the origin, where they stick to a growing cluster, following an electrostatic law (see FIG. 3). This celebrated Diffusion-Limited Aggregation model is still not understood theoretically, in spite of having been heavily investigated numerically in the past 25 years. The two problems share an important feature, belonging to the singular perturbation class of models. While this explains the very intricate patterns observed experimentally and numerically, it also makes theoretical explorations particularly difficult.

Together with Paul Wiegmann (U. of Chicago) and A. Zabrodin (ITEP, Moscow) we have developed an analytical method for regularization applicable to both problems, and based on a complex extension of Random Matrix Theory [2–4]. The evolution of the system is then found to be governed by an integrable nonlinear differential equation, determined entirely by the type of boundary singularity, which explains the universality features observed.

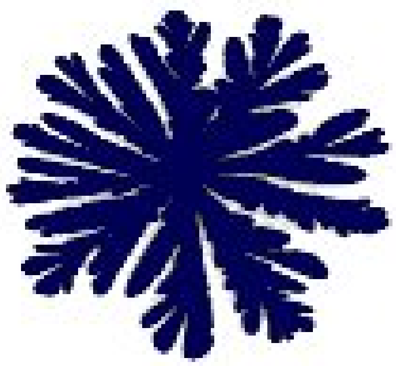

FIG. 2: Laplacian Growth pattern.

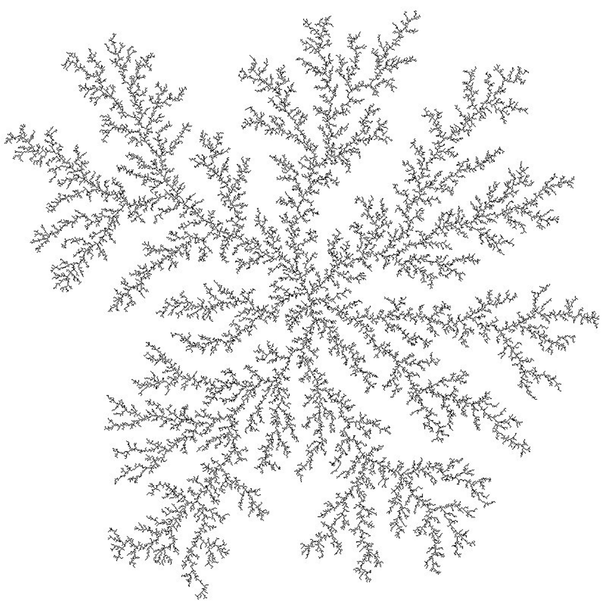

FIG. 3: Diffusion Limited Aggregation cluster.

C. Non-equilibrium coherent quantum dynamics

Developed during the last five years, systems of cold fermionic gases held in optical traps have become the most accurate physical devices exhibiting BCS superconductivity. Unlike in traditional solid-state superconductors, the time scales associated with various phenomena specific to superconductivity (the order parameter and quasiparticle time scales) have magnitudes which allow the observation of rich, nonlinear coherent quantum dynamics.

A full theoretical understanding of these phenomena is far from complete, while particular questions of physical importance have been answered [5]. A continuation of these studies is underway [6], with emphasis placed on possible applications of these nonlinear quantum phenomena. Such applications may include quantum coding/decoding of information, novel data storage tools, and a new class of quantum interferometers.

D. Mathematical methods for complex physics

A recurrent theme for all the systems described is the lack of appropriate analytical tools, as opposed to "reducible" physical systems. This is due to considerable difficulties one encounters when trying to solve the existing models. We are studying such relevant models with Mihai Putinar (UCSB) at the interface between functional analysis, algebraic geometry, and random matrices [7], with A. A. Abanov (SUNY) and Paul Wiegmann regarding nonlinear bosonization of interacting electrons and their relation to integrable hierarchies, and with Eli Ben-Naim (T-13) concerning nonlinear dynamics of selection processes in statistical physics.

Another project to be pursued with Misha Chertkov, G. Falkovich (Weizmann Institute) and collaborators is related to recent numerical observations of invariant structures in 2D turbulence, and their relation to quantum critical phenomena.

[1] Image credit: NASA.

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[7] M. Mineev, M. Putinar, R. Teodorescu, P. Wiegmann, *J. Phys. A: Math. Gen.*, in preparation.